An Analysis of Pulse Shape and Evolution

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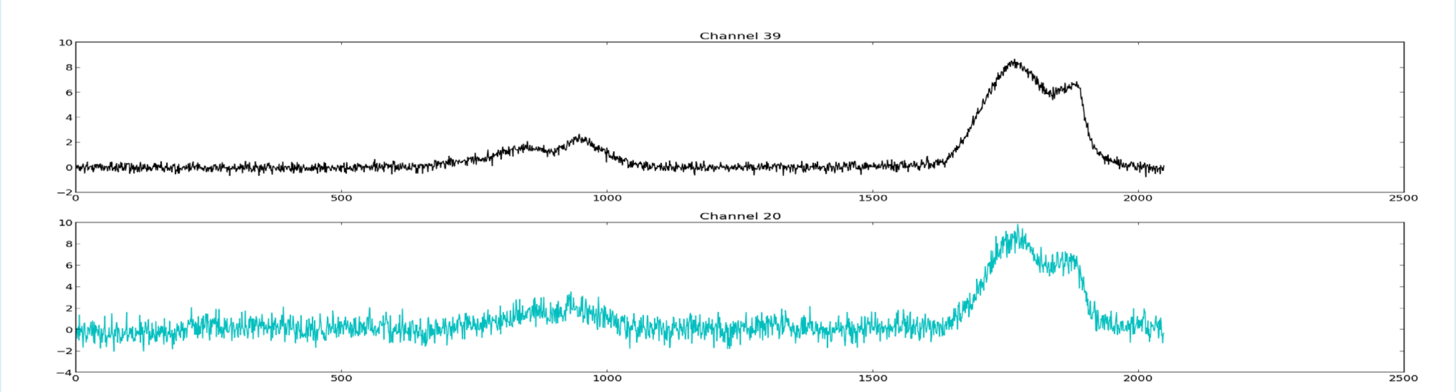
Purpose

Using data collected from Arecibo and Green Bank, we analyzed the shape and evolution of pulses from various millisecond pulsars. Each pulsar has a very unique pulse shape that varies over frequencies ranging from 400 MHz to 2000 MHz. Our goal was to create a mathematical model for the variation of pulse profile over frequency. We utilized Python and various modules therein to accomplish this. The ultimate goal of this project and NANOGRAV as an entirety is to time these pulsars to precisions of 10 nanoseconds in order to detect small fluctuations in the time of arrival of pulses, which could indicate the presence of a gravitational wave. In order to attain NANOGrav's goal of detecting gravitational waves, we must measure pulse arrival times as precisely as possible. These measurements are made by fitting the pulse shape from any given observation against a mathematical model of the pulse shape which is shifted back and forth to find the best fit. Having the best possible mathematical model is critical to this process.

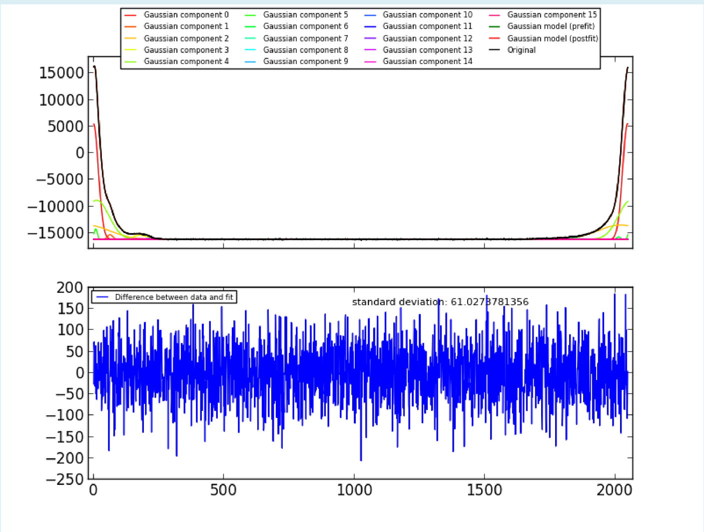


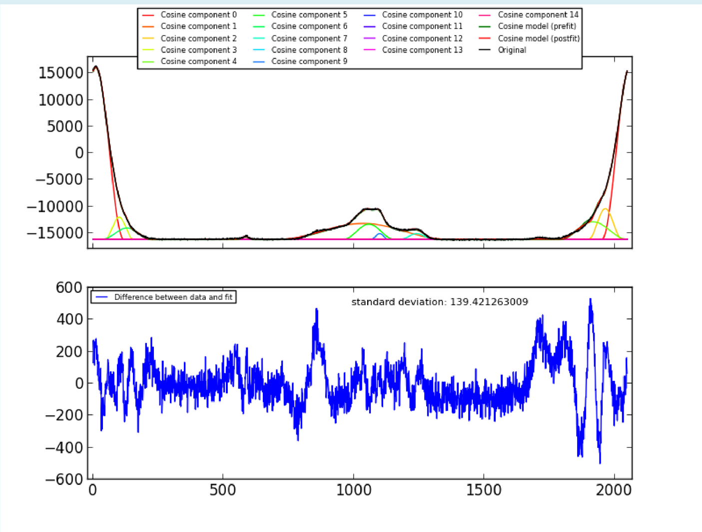
Example of pulse shape evolution for pulsar B1855+09

Gaussian, Lorentzian, and Cosine Models

Initially, we attempted to model the pulse profiles using an analytical approach. We began by trying to model certain frequency channels using Gaussian, Lorentzian, and cosine curves, which contain four parameters each. We wrote various programs in Python that utilized matplotlib, pylab, and scipy.optimize to fit these curves to our data sets. We started by writing programs that required a manually input initial guess for each parameter within the various curves. We eventually determined a way to have Python compute initial parameters, including the offset level of the profile, the full width of the peak, the value where the peak reaches its maximum, and the standard deviation from that value. Our programs fit an indicated number of Gaussian components to the data and plotted the results, as well as the data minus the model generated by the superposition of these curves. The results showed us that using these types of curves was not an effective means of modeling the profiles. Each pulse shape required at least 15 curves to efficiently represent the data, which meant that over 60 parameters were needed, but even this was not always sufficient to represent the pulse shape with complete accuracy.

Examples of plots generated by our programs. Top Left: Gaussian approximation of J1713+0747 Top Right: Cosine approximation of (pulsar name) Bottom Right: Lorentzian approximation of B1953+29



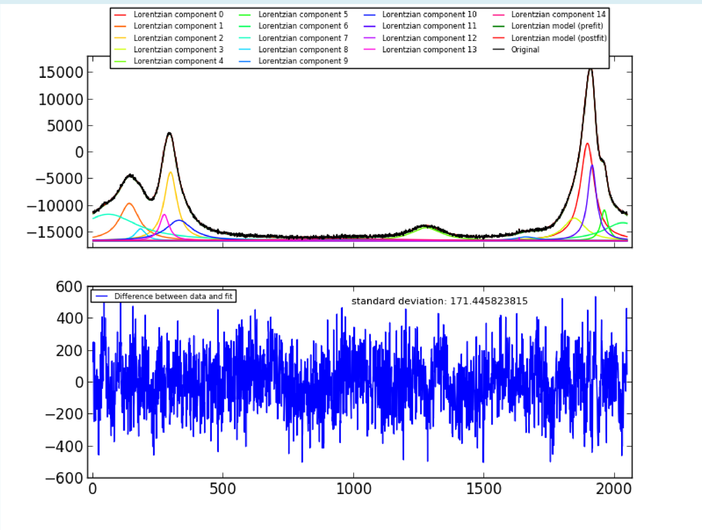


Lorentzian

$$y = y_0 + \frac{2A}{\pi}\frac{w}{4(x-x_c)^2+w^2}$$

Gaussian

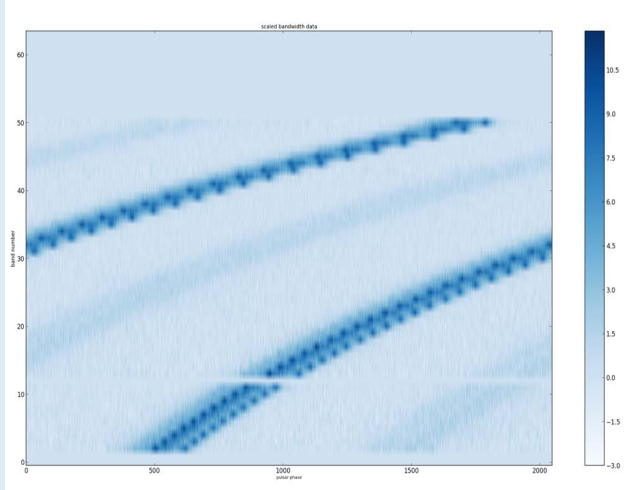
$$y = y_0 + \frac{A}{w\sqrt{\pi/2}}e^{-2\frac{(x-x_c)^2}{w^2}}$$



Dispersion Measure

Signals from pulsars must travel through the interstellar medium before reaching Earth. The interstellar medium is comprised of electrons and other forms of matter that disperse the pulsar signals before they reach our telescopes. Each pulsar's signal must be "de-dispersed" to remove this affect from the data and align pulses over different frequencies so that they appear to arrive simultaneously. Doing this requires knowing the dispersion measure of the pulsar, which is the integral of the electron density of the interstellar medium between the pulsar and the Earth. We wrote a program that determined the best dispersion measure for each data file using optimization tools. We then used this dispersion measure to correctly align the frequency channels and utilized these corrected data files for our pulse-shape analysis.

Dispersed data from J0030+0451

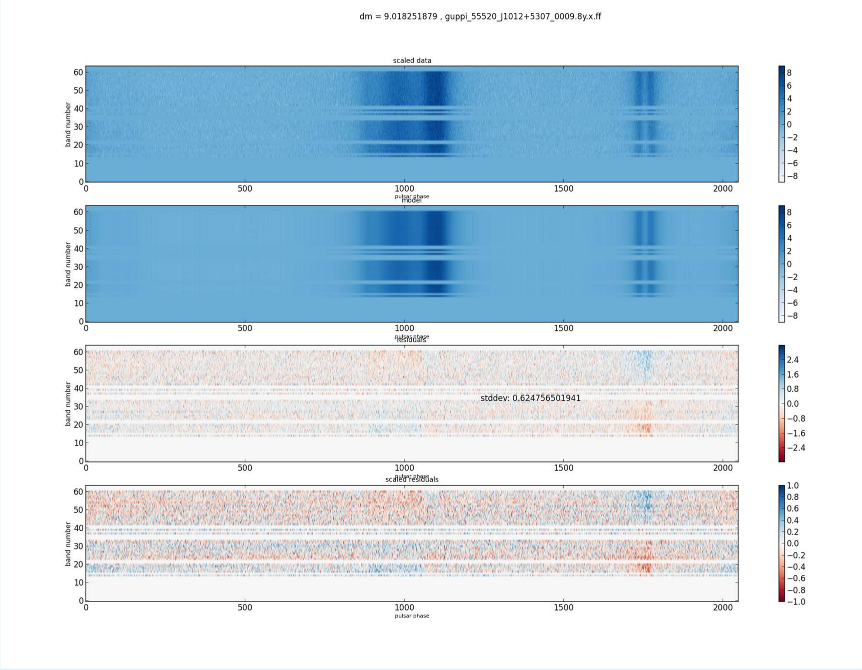


Taylor Polynomial Model

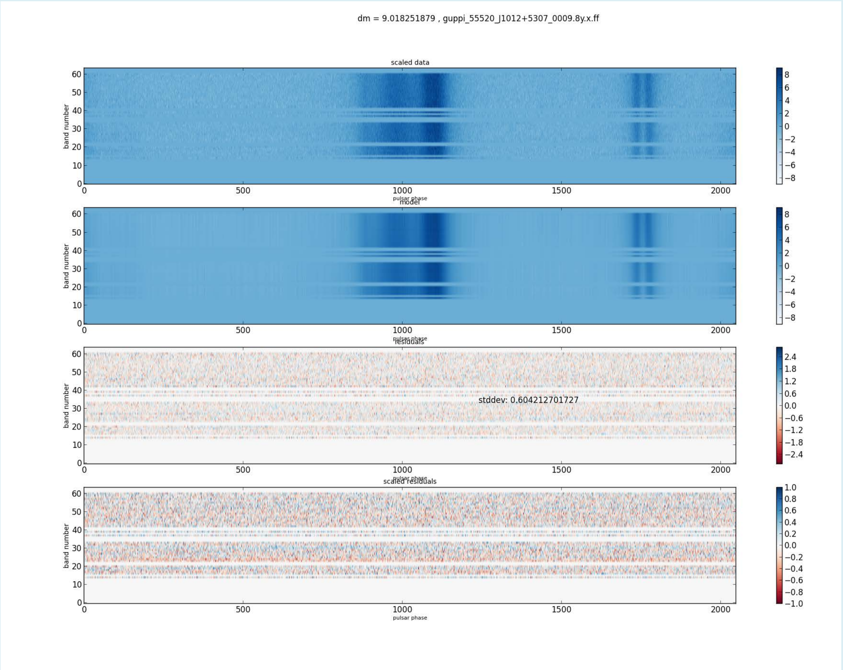
We found that the Gaussian-component method produced complicated models that were not always fully accurate, so we adopted a different approach, using Taylor expansions to represent the pulse profiles. More specifically, if y(t,f) is the pulse profile at time t (where t goes from 0 to the pulse period) and radio frequency f, then we expanded it as $y(t,f) = y_0(t) + y_1(t)\,(f-f_0) + (1/2)y_2(t)\,(f-f_0)^2 + \ldots$ where $y_0(t)$, $y_1(t)$, $y_2(t)$, etc., are the average profile, its linear variation, its quadratic variation, etc., and where f_0 is the center frequency. We wrote python code to perform this expansion. The program determines an average profile and then makes corrections for each frequency channel as the pulse shape varies. Our goal was to determine if there was one particular order that sufficiently modeled each and every profile. We concluded that for a majority of the pulsars, first order approximations sufficed.

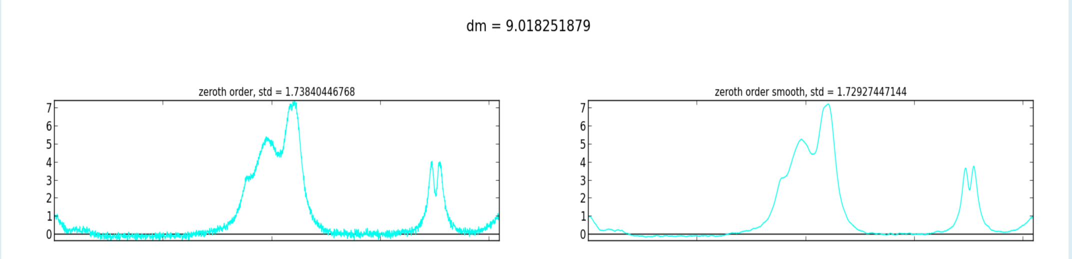
These models also allowed us to clearly see profile evolution over the observed radio band. In the plots below, one can see clear stripes of red and blue, indicating differences from the average pulse shape as frequency changes.

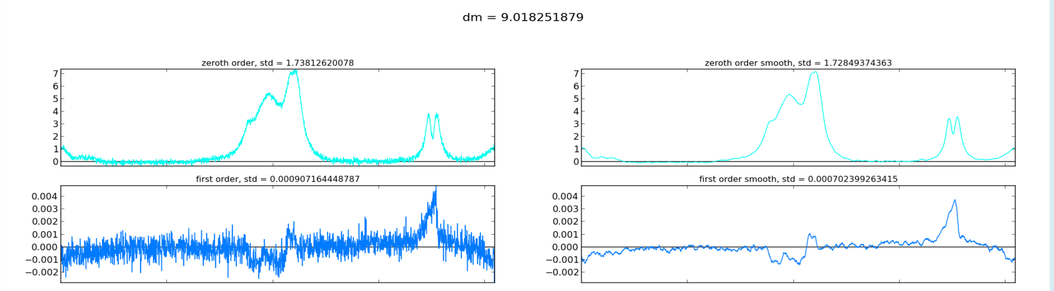
Zeroth order approximation of J1012+5307



First order approximation of J1012+5307







Upper plots: The top subplot is a graph of the data that was collected. The second subplot is a graph of the Taylor approximation of our data. The third plot is a graph of the data minus the Taylor model, which allows us to see significant differences between our approximation and the actual signal. The last plot is essentially an enhanced version of the third plot; we merely took the residual plot and rescaled each frequency channel so that the area under the residual curve was uniform. For each plot, the vertical axis is radio frequency; some radio frequency channels are missing because of weak pulsar signals and/or radio frequency interference.
Bottom plots: These are the Taylor approximations generated by our program, the left being the zeroth approximation, and the right being the linear approximation. Each of these plots has the actual approximation and a smoothed version thereof.