Millisecond Pulsar Timing 2: Radio Frequency Dependent Timing Delays

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Interstellar Dispersion

Lower-frequency radio waves travel through the ISM more slowly than high frequency waves, causing them to reach Earth slightly later. This effect is called dispersion, and results in a time delay:

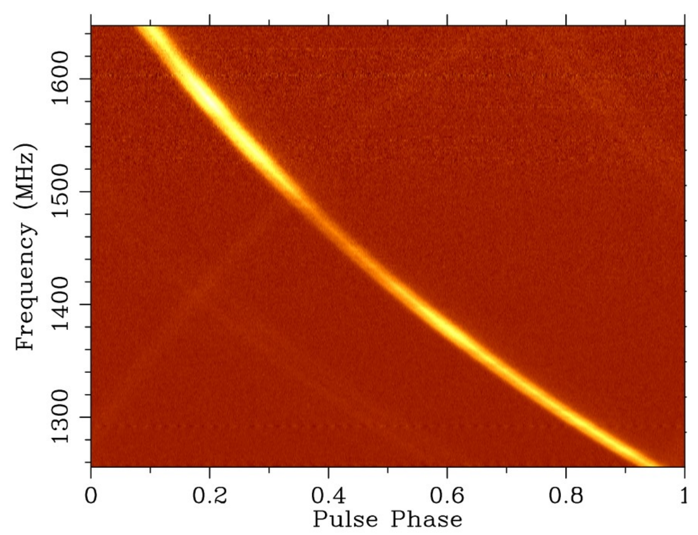
$$\Delta t = \mathcal{D} \times \frac{\mathrm{DM}}{f^2}$$

Where *f* is the radio frequency, D is a constant and DM is a measured quantity called the dispersion measure. The DM is defined as:

$$\mathrm{DM} = \int_{\mathrm{Earth}}^{\mathrm{Pulsar}} n_e \mathrm{d}l$$

Where n_e is the electron density of the ISM.

By observing delays in the pulse TOAs, we can calculate the dispersion measure and subtract the effects from our data. The concentration and composition of the ISM varies, so as the pulsar moves through it, the DM changes. We measure the DM as it changes over time – this is called the DMX.



Above: This figure shows the change in pulse arrival time across different frequencies due to dispersion.

Acknowledgements

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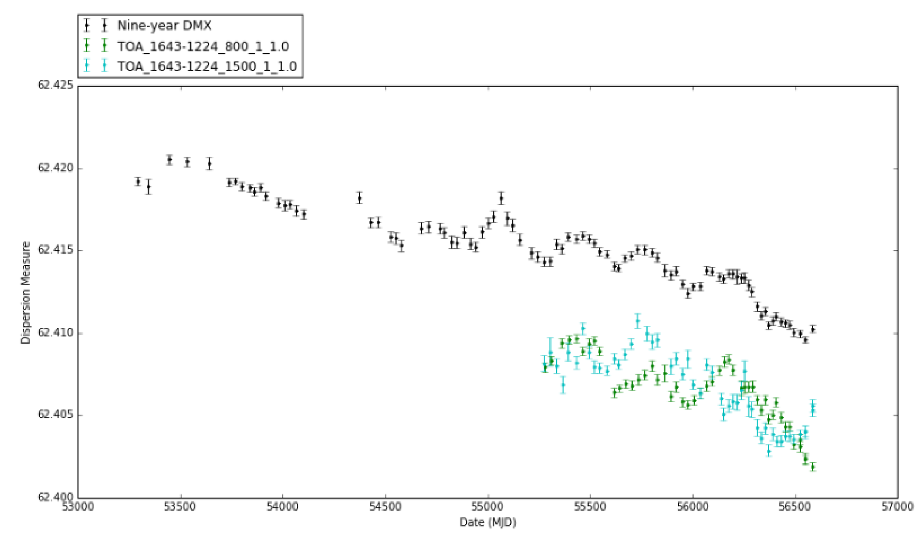
Problems With Previous Methods



In Hao Lu's ('16) senior thesis, he found that for some pulsars, the DMX varies differently at different frequencies. This suggests that there is some other frequency dependent phenomenon affecting the TOA.

Our research this summer focused on trying to characterize this phenomenon mathematically and astrophysically.

Above: This figure is from Hao Lu's thesis. It illustrates dispersion measure over time for the pulsar J1643-1224. The blue and green points correspond to DM measurements calculated using different frequency ranges. The green data are from the lower frequency band while the blue data correspond to higher frequencies.

Our Methods

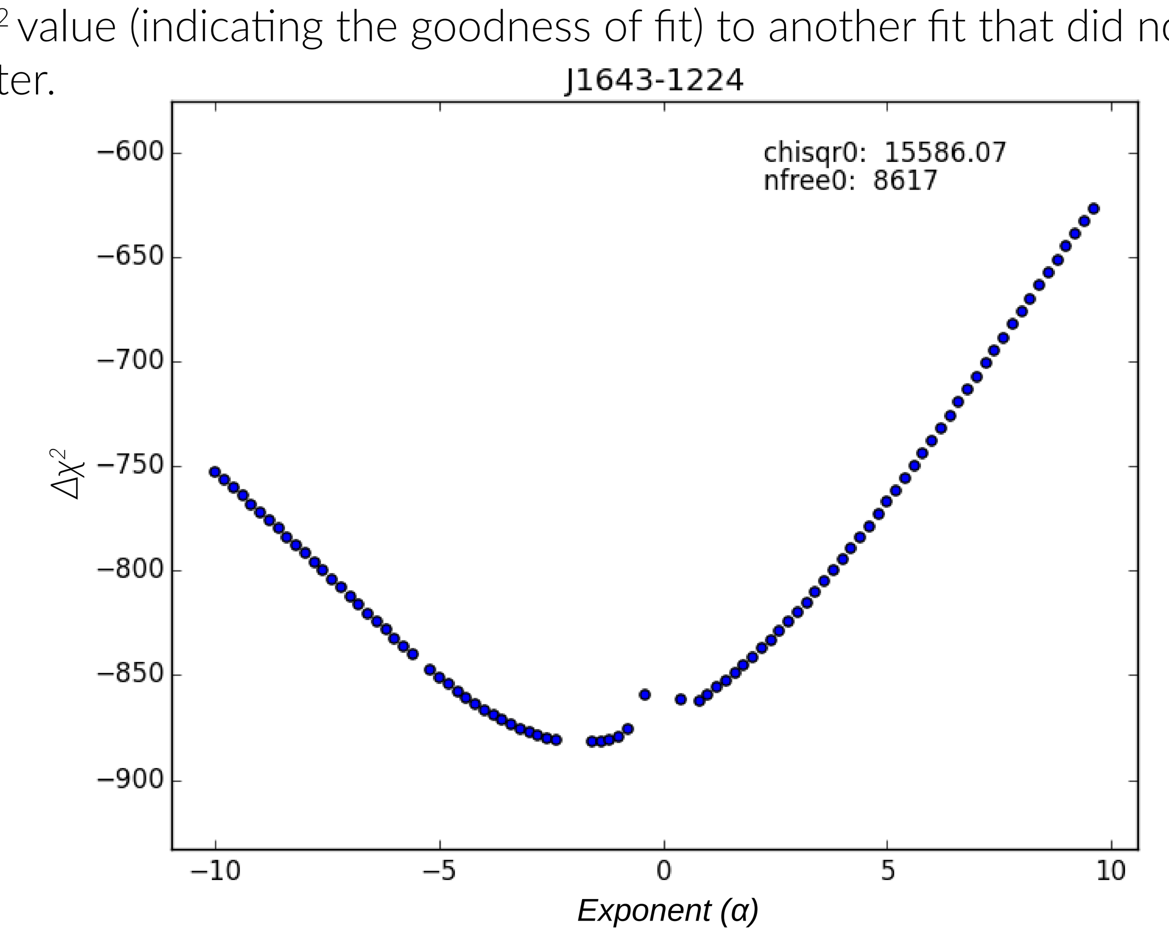
We started by introducing a new, frequency-dependent parameter to our model and seeing if it improved the precision of the model. We called this new parameter "XMX." At first, we theorized that XMX could cause a time delay that is proportional to the frequency raised to some exponent. We tested many different exponents for each pulsar, and compared the fit for each. Below is a complete version of the equation for $t_{observed}$:

$$t_{\mathrm{observed}} = t_{\mathrm{emitted}} + \underbrace{\Delta t_{\mathrm{solar\ system}} + \Delta t_{\mathrm{binary}}}_{\text{Distance-related components}} + \underbrace{\mathrm{DMX}_i\left(\frac{\mathcal{D}}{f^2}\right)}_{\text{Old ISM model}} + \underbrace{\mathrm{XMX}_i\left(\frac{f}{f_0}\right)^{\alpha}}_{\text{Our addition to the model}}$$

- We analyzed data from 44 pulsars from NANOGrav's 11-year dataset to provide experimental TOAs.
- We fit timing models incorporating the XMX parameter to the data.
- Our software predicted TOAs and compared them to the experimental data; the difference between the two is called a "timing residual."
- We then compared the χ^2 value (indicating the goodness of fit) to another fit that did not include the XMX parameter.

Right: The resulting χ^2 plot for the pulsar J1643-1224. On the x-axis is the exponent of the XMX parameter. On the y-axis is the $\Delta\chi^2$ value.

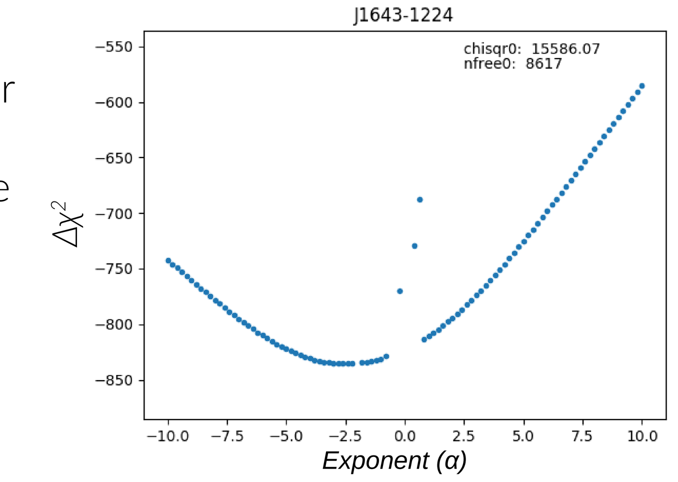
A lower χ^2 value corresponds with an improvement to the model precision when we introduce a parameter with that exponent.

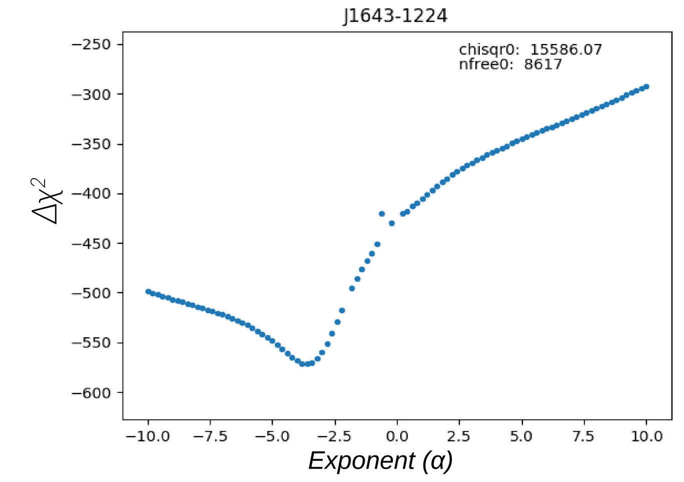


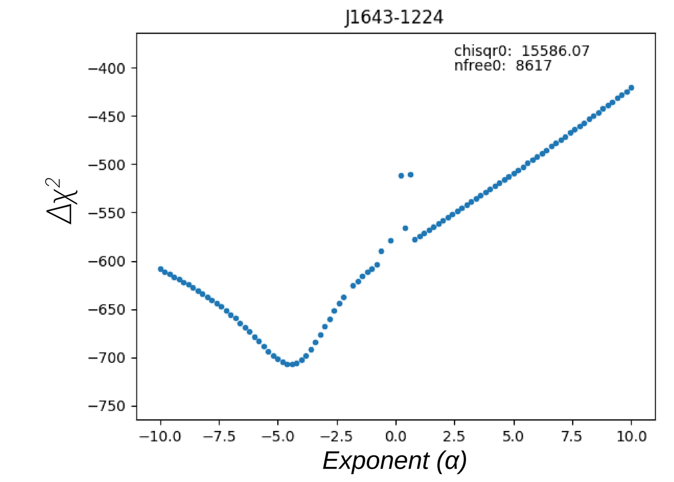
XMX Varying Timescales

We also tried varying the timescale over which XMX was calculated to see if we could observe broader trends over longer periods of time. These three plots are all χ^2 plots for the pulsar J1643-1224 where the XMX is calculated every 50, 100, or 200 days instead of every 6

Top Left: 50 days.
Bottom Left: 100 days.
Bottom Right: 200 days.



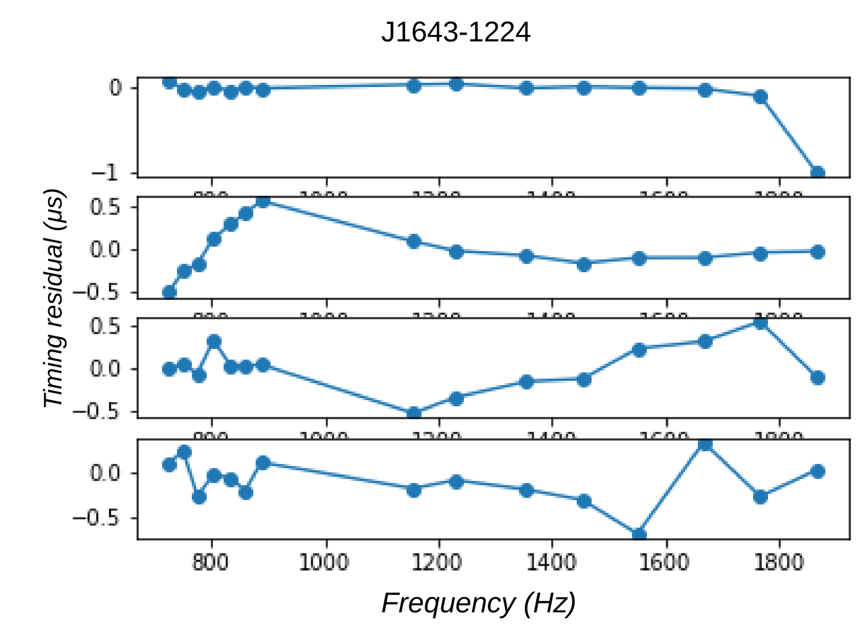




Principal Component Analysis and Future Work

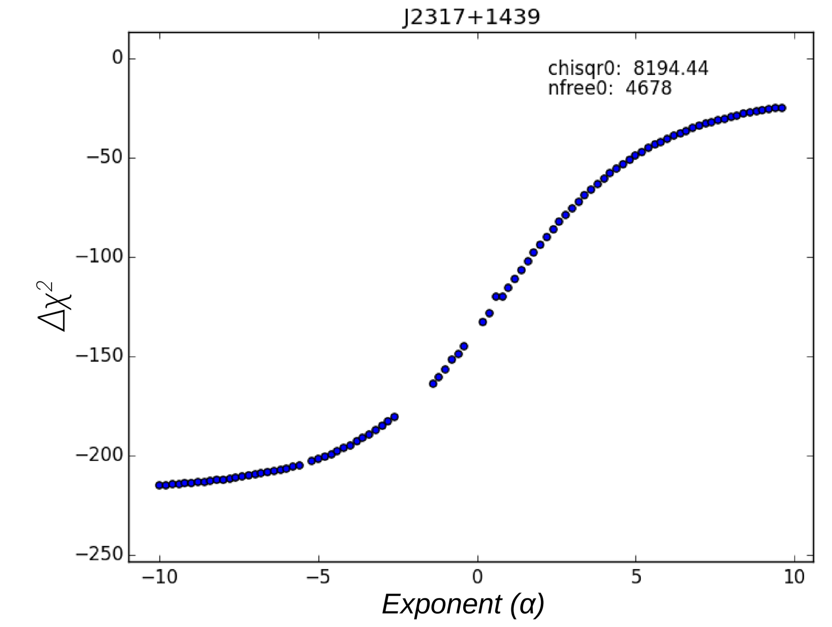
Principle Component Analysis (PCA) is a statistical tool that we used. It is used to analyze trends in large sets of data. We used it to try to determine how the residuals changed with frequency on average over time. Our reason for doing this was to help look for frequency-dependent parameters that are not related by a power law to the frequency. Due to time constraints, we have yet to thoroughly analyze the results; however, we believe it may be useful for this research to be continued.

In addition to this, some of the variations in the XMX plots seemed to show a significant improvement in the model for lower exponents. We are unsure as to why this might be the case, but believe that it merits further study.



Above: A plot generated in our PCA of pulsar J1643-1224 showing the first four principle eigenvectors.

Below: A χ^2 plot for the pulsar J2317+1439, showing the "trailing off" behavior in lower exponents.

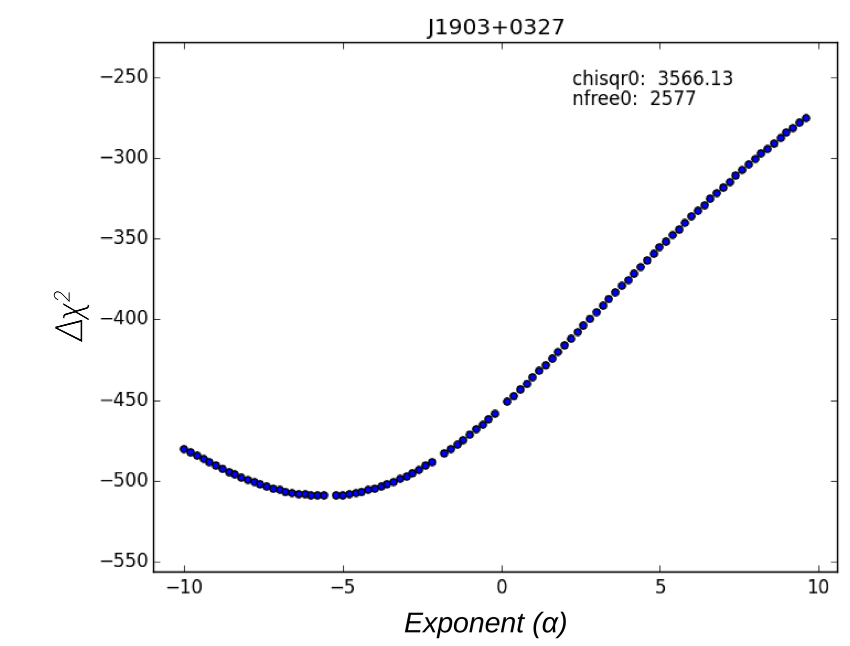


Results

Through this method, we generated a variety of different plots, leading us to believe that there may be different causes in frequency variations for different pulsars. Below are three of our plots demonstrating some of the patterns of behavior we observed, along with analyses.

Several of the pulsars we observed exhibited minima around the exponent -4. We believe that this may be characteristic of scattering, as scattering is known to delay radio waves in proportion to their frequency to the -4^{th} power.

Scattering is and effect caused by the refraction of the pulses in the interstellar medium. It causes time delays that impact our model, but is not observed in all of our pulsars. This makes sense, as the ISM is not homogeneous throughout space. From these plots, we can tell which pulsars appear to have scattering based on where there minima lie.



Above: J1903+0327. This pulsar appears to have a minimum around -4.

Below: J1600-3053, another good example of probable scattering.